Role of Various Entropies in the Black Hole Information Loss Problem

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In a recent paper Hawking has argued that there is no information loss in black holes in asymptotically AdS spacetimes. We remind that there are several types of information (entropy) in statistical physics – fine grained (microscopic) and coarse grained (macroscopic) ones which behave differently under unitary evolution. We suggest that the coarse grained information of the rest of the Universe is lost while fine grained information is preserved. A possibility to develop in quantum gravity an analogue of the Bogoliubov derivation of the irreversible Boltzmann and Navier - Stokes equations from the reversible mechanical equations is discussed.

INTRODUCTION

In 1976 Hawking has argued that the black hole creation and evaporation could lead to an evolution of a pure state into a mixed state, which is in contradiction with the rules of quantum mechanics [1]. This has become known as the black hole information loss problem (or black hole information paradox), for a review see [2].

In a recent paper Hawking [3] has suggested that there is no information loss in black holes in asymptotically AdS (anti-de Sitter) spacetimes. The central point in his argument is the assertion that there is no information loss if one has a unitary evolution. Then, since the AdS quantum gravity is dual to a unitary conformal field theory [4, 5, 6], there should be no information loss.

In more detail the proposal looks as follows. Black hole formation and evaporation is considered as a scattering process when all measurements are made at infinity. The evolution operator is defined by means of the path integral for the partition function. In the sum over topologies in the path integral there are trivial topologies, which lead to the unitary evolution, and nontrivial black hole topologies. Information is lost in topologically non-trivial black hole metrics, but their contribution decays to zero at large times, at least for each separate mode,[7]. As a result one gets a unitary evolution at large times and the information is preserved in that limit.

We make two remarks concerning this proposal.

(i) Actually it is proposed that we can make observations only at such large time scales where small black holes either do not form yet or have already evaporated. In any case black holes are not present then.

(ii) An important point in the argument is the assertion that, if one considers a theory with a unitary dynamics, there is no information loss problem.

Concerning the point (i) there is a question of how information gets out of a black hole. In fact it is one of the main questions in the whole discussion of the information paradox. Hawking's answer is that there is no sense in asking this question because its answer would require to use a semi-classical metric, which has already lost the information. We discuss this argument below.

About the point (ii) we remind that the problem of how a unitary reversible dynamics can lead to an irreversible behavior (i.e. to information loss) is the subject of numerous studies in statistical physics. One specific application is the problem of relaxation: Even though quantum evolution, believed to describe all condensed matter, is unitary, we observe relaxation in every day life [9]. It is thus clear that much of the irreversible behavior, including probably the expansion of the Universe, should be compatible with unitary quantum mechanics. Point (ii) is the main point which will be discussed in this note.

We consider the black hole information problem as a particular example of the fundamental irreversibility problem in statistical physics. We point out that similar problem occurs when we study ordinary gas or the formation of the ordinary black *body* and its thermal radiation. Actually, one has to give a quantum mechanical explanation for the

emergence of the second law of thermodynamics in macroscopic systems.

The irreversibility problem was much studied by Boltzmann and many other authors. There is not yet a complete solution but a deep understanding of the problem has been achieved [8] - [17].

Information is usually quantified as entropy with a minus sign. There are two classes of entropies in statistical physics – the fine grained (microscopic) one and several coarse grained (macroscopic) ones. They behave differently under unitary evolution. For a given model, unitary dynamics might increase one or more coarse grained entropies while preserving the fine grained one. Whether there is an increase of a certain coarse grained entropy (i.e. loss of *this* information) is a dynamical question and its answer comes for a given model from a thorough investigation of its dynamics, which should show a sort of instability or ergodicity and mixing. Alternatively, coupling to a bath would suffice. This is relevant for the black hole situation, where the "bath" is the rest of the Universe.

We note that the properties of the coarse grained (in particular thermodynamical) entropy are important in statistical physics while the fine grained entropy is not so significant since typically it cannot be determined and, remaining conserved, would not reflect a specification of the dynamical evolution.

DIFFERENT KINDS OF ENTROPIES

Let us split up our degrees of freedom in two classes, those of the black hole and those of the rest of the Universe. The latter we call "bath", since for a proper formulation of thermodynamics of black holes, the rest of the Universe indeed plays the role of the thermal bath in condensed matter problems. The density matrix of the total system ρ has several marginals. The reduced density matrix of the black hole is

$$\rho_{\mathrm{BH}} = \mathrm{Tr}_{\mathrm{B}}\, \rho$$

while the reduced density matrix of the bath is

$$\rho_{\mathrm{B}} = \mathrm{Tr}_{\mathrm{BH}}\, \rho$$

This brings us three von Neumann entropies: the "fine grained" entropy of the total system,

$$S_{\mathrm{total}}^{\mathrm{fine}} = -\mathrm{Tr}\, \rho\, \ln \rho.$$

This quantity is conserved in time for unitary motion. If one starts out from a pure state of incoming matter and bath, it vanishes at all times. Next there is the fine grained von Neumann entropy of the black hole,

$$S_{\mathrm{BH}}^{\mathrm{fine}} = -\mathrm{Tr}_{\mathrm{BH}}\, \rho_{\mathrm{BH}}\, \ln \rho_{\mathrm{BH}}$$

and the fine grained von Neumann entropy of the bath,

$$S_{\mathrm{B}}^{\mathrm{fine}} = -\mathrm{Tr}_{\mathrm{B}}\, \rho_{\mathrm{B}}\, \ln \rho_{\mathrm{B}}$$

When starting from a pure state, entangled or not, the latter two will be equal at all times. Though then vanishing at $t = 0$ they become positive at later times, $S_{\mathrm{BH}}^{\mathrm{fine}}(t) = S_{\mathrm{B}}^{\mathrm{fine}}(t) > 0$. At large times one expects them to go to zero again [22]. For $S_{\mathrm{BH}}^{\mathrm{fine}}$ the reason is simply that matter is radiated, making it smaller and smaller, so that with the matter its entropy evaporates. For $S_{\mathrm{B}}^{\mathrm{fine}}$ it theoretically expected, because, in the final absence of the hole, it just reflects the purity of the state. But physically it is a surprising and counter-intuitive result, since this vanishing entropy clearly does not reflect the energy radiated into this bath by the black hole evaporation.

Coarse graining can be done at larger and larger scales. Thus there is still another entropy to consider, namely the coarse grained entropy of the bath, where one neglects all correlations between bath modes. One first has to define the reduced density matrix of a given mode, $\rho_{\mathrm{mode}} = \mathrm{Tr}_{\mathrm{all\ other\ modes}}\, \rho$ and from it the von Neumann entropy

$$S_{\mathrm{B}}^{\mathrm{coarse}} = \sum_{\mathrm{mode}} S_{\mathrm{mode}} = \sum_{\mathrm{mode}} -\mathrm{Tr}_{\mathrm{mode}}\, \rho_{\mathrm{mode}} \ln \rho_{\mathrm{mode}}$$

This is the one entering quasi-classical discussions of the black hole information paradox. Since it probably does not vanish at large times, it is a candidate for our "natural" association of entropy with a measure of disorder. It is then obvious that $S_{\mathrm{B}}^{\mathrm{fine}}$, which does vanish at large times, takes the correlations between different modes, neglected in $S_{\mathrm{B}}^{\mathrm{coarse}}$, into account in a very subtle manner.

Boltzmann's famous formula for the entropy reads

$$S = k_B \log W$$

The W in this formula is the number of microstates compatible with the *macroscopic* state. Therefore this formula defines a coarse grained entropy. There is a remarkable computation by Strominger and Vafa of the Bekenstein - Hawking coarse grained entropy by counting of microscopic BPS states in string theory [18].

Black hole thermodynamics is a problem with two temperatures: the Hawking temperature of the hole and the $3K$ back ground temperature [19] [20]. These distinctions automatically show up in condensed matter analogs of the black hole evaporation problem, that we plan to discuss elsewhere [21].

The thermodynamic entropy of a system in contact with a bath at temperature T is defined in terms of added heat dQ by the Clausius inequality

$$dQ \leq TdS \tag{1}$$

taken as an equality, so $dS = dQ/T$. The "classical intuition" of entropy arose when Bolztmann showed that this thermodynamic entropy agrees with his measure of disorder, more precisely, the logarithm of the number of states. For a closed system, this quantity cannot decrease. For systems with two temperatures a generalized Clausius inequality may hold when there are also two well separated time scales, leading to two different entropies, that enter as: $dQ \leq T_1 dS_1 + T_2 dS_2$. This applies to glasses [23] and black holes [19].

For nanoscopic and mesoscopic systems, one has led to uncover the field of "quantum thermodynamics" [24].

Page has discussed the microcanonical and canonical entropies of black holes [25]. The first one does not reflect its environment, the second one assumes it to be at the Hawking temperature, which is the case only for a specific black hole size and, moreover, an unstable situation.

We have discussed various entropies which are used to describe the *classical* capacity of a quantum channel. To describe the *quantum* channel capacity the quantum mutual entropy [26] and the quantum coherent information [27] are used. It would be interesting to investigate the role of these entropies in quantum gravity.

ON THE BLACK HOLE INFORMATION PARADOX

Our proposal for the investigation of the black hole information loss problem is the following. One of the mentioned entropies is the coarse grained bath entropy. It increases during the evolution. In this sense there is information loss. But it does not mean that there is loss of the fine grained information (entropy). The whole picture of the black hole formation and evaporation is similar to the formation and radiation of a black body.

In a specific model the increase of coarse grained entropy has to be demonstrated if the black hole evaporation indeed behaves as a thermodynamic problem.

The next step in the program is to develop in quantum gravity an analogue of the Bogoliubov derivation of the Boltzmann and Navier - Stokes equations from the Liouville equation [8, 10, 11]. We shall sketch that in the last section. In quantum field theory one derives quantum stochastic differential equations [12]. But we should stress that a much better understanding of the irreversibility problem in various models is required.

It would also be interesting to investigate the role of the quantum mutual entropy [26] and of the quantum coherent information [27] in quantum gravity.

INFORMATION LOSS IN GASES

Consider a classical or quantum gas of N particles in a box of the volume V which is described by the Hamiltonian

$$H_N = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} \Phi(\mathbf{x}_i - \mathbf{x}_j).$$

Here $\mathbf{x}_i$ are positions, $\mathbf{p}_i$ are momenta, m is mass and $\Phi(\mathbf{x})$ is the interaction potential between a pair of particles.

One has a reversible classical dynamics and a unitary quantum dynamics. Hence, in neither situation there is a loss of the fine grained information.

However, it is well known that the kinetic theory of gases is based on the Boltzmann equation which reads [10, 11]

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \frac{\partial f}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = J(f). \tag{2}$$

Here $f = f(\mathbf{x}, \mathbf{p}, t)$ is the one-particle distribution function, t is time, $\mathbf{F} = \mathbf{F}(\mathbf{x}, t)$ is the force, and $J(f)$ is a bilinear functional in f.

The coarse grained Boltzmann entropy is defined by

$$S_B(t) = -\int f(\mathbf{x}, \mathbf{p}, t) \ln f(\mathbf{x}, \mathbf{p}, t) d\mathbf{x} d\mathbf{p}.$$

Boltzmann has proven (H-theorem) that

$$\frac{dS_B(t)}{dt} \geq 0$$

Moreover the entropy is constant only for an equilibrium distribution function. For a non-equilibrium state the Boltzmann entropy increases and therefore one gets *information loss.*

Now one can ask the same question as for the black hole information loss problem. How is it possible that one gets information loss and irreversibility for a system of N particles which is described by a reversible dynamics?

An important progress in investigation of this question was achieved by Bogoliubov [8, 10, 11]. He considers a system of equations for s-particle correlation functions

$$f_s(\xi_1, ..., \xi_s, t)$$

where $\xi_i = (\mathbf{x}_i, \mathbf{p}_i)$, $i, s = 1, 2, ..., N$. The system of equations (BBGKI-chain) is equivalent to the Liouville equation and is reversible.

Then for the dilute gases Bogoliubov introduces a kinetic relaxation time scale τ_0 and uses the thermodynamical limit ($N, V \to \infty$, $N/V = $ const) and the factorization of the s- particle correlation functions f_s in terms of the one particle distribution function f:

$$f_s(\xi_1, ..., \xi_s, t) \to \prod_{i=1}^{s} f(\xi_i, t), \quad t > \tau_0, \quad s = 2, 3, ..., N \tag{3}$$

In this way he was able to obtain the Boltzmann equation. Then one can use the Boltzmann equation to derive the hydrodynamical Navier - Stokes equation.

We write the Boltzmann equation (2) symbolically as

$$\frac{\delta f}{\delta \sigma} = J(f) \tag{4}$$

where $\sigma = (t, \xi)$.

Bogoliubov used a similar approach also for derivation of quantum kinetic equations. In this case one uses the correlation functions

$$f_{sk}(\mathbf{x}_1, ..., \mathbf{x}_s; \mathbf{y}_1, ..., \mathbf{y}_k; t) = Tr[\rho_t \psi(\mathbf{x}_1)...\psi(\mathbf{x}_s)\psi^+(\mathbf{y}_1)...\psi^+(\mathbf{y}_k)]$$

Here ρ_t is the density operator at time t and $\psi(\mathbf{x})$, $\psi^+(\mathbf{y})$ are annihilation and creation operators satisfying the usual commutation relations $[\psi(\mathbf{x}), \psi^+(\mathbf{y})] = \delta(\mathbf{x} - \mathbf{y})$. To derive quantum kinetic equation one uses an approximation similar to the Bogoliubov approximation (3). It is a kind of the mean field approximation.

There are other models in which an irreversible behavior from reversible dynamics was derived, [12, 13]. Mathematical studying of these questions is also the subject of ergodic theory where for certain dynamical systems the properties of ergodicity and mixing were established and where the notions of classical and quantum Anosov and K-systems and the Kolmogorov - Sinai entropy play an important role [14, 15, 16, 17].

It was demonstrated by Pauli, von Neumann, van Hove, and Prigogine that in quantum mechanics the coarse grained entropy increases as a result of the unitary dynamics.

The transition from the BBGKI chain of equations for the family of correlation functions $\{f_s\}$ to the Boltzmann equation for the one particle distribution function f is the transition from the fine grained reversible description to the

irreversible coarse grained description. We loose information when we describe a gas by means of only the one-particle distribution function $f(\mathbf{x}, \mathbf{p}, t)$, or hydrodynamical variables, which are integrals of f with some weights.

In quantum gravity we interpret the distribution $f[g, \pi]$ of the classical metric $g_{\mu\nu}(x)$ and its conjugate $\pi_{\mu\nu}(x)$ as an analogue of the one particle distribution $f(\mathbf{x}, \mathbf{p}, t)$ or hydrodynamical variables. To get insight into the black hole information loss problem, one has to develop in quantum gravity an analogue of the Bogoliubov derivation of the Boltzmann equation from the Liouville equation. In quantum field theory one derives quantum stochastic differential equations [12].

INFORMATION LOSS IN QUANTUM GRAVITY

One can use the Euclidean [3] or the Wheeler - De Witt [28] approach to quantum gravity. Each one has its own advantages and disadvantages. One of problems with the Euclidean approach is that one can define there the Green functions and the partition function but not the scattering matrix. Moreover, path integrals are convenient to write down semiclassical expansion but one can not use this "sane" formalism to solve spectral problems, even to compute the spectrum of the hydrogen atom.

The transition amplitude between configurations of the three-metric h'_{ij} and field Φ' on an initial spacelike surface Σ' and a configuration h''_{ij} and Φ'' on a final surface Σ'' is

$$< h'', \phi'', \Sigma'' | h', \phi', \Sigma' > = \int e^{\frac{i}{\hbar} S[g, \Phi]} \, \mathcal{D}\Phi\mathcal{D}g,$$

where the integral is over all four-geometries and field configurations which match given values on two spacelike surfaces, i.e. $\Phi|_{\Sigma'} = \phi'$, $g|_{\Sigma'} = h'$, $\Phi|_{\Sigma''} = \phi''$, $g|_{\Sigma''} = h''$.

The problem of creation of black holes in quantum theory is considered in [29] - [32]. The role of boundary conditions in the path integral describing the creation of black holes is discussed in [31].

We are interested in the process of black hole creation. Therefore Σ' is a partial Cauchy surface with asymptotically simple past in a strongly asymptotically predictable space-time and Σ'' is a partial Cauchy surface containing black hole(s), i.e. $\Sigma'' - J^-(\mathcal{T}^+)$ is non empty.

Black holes are conventionally defined [33] in asymptotically flat (or AdS) space-times by the existence of an event horizon H. The horizon H is the boundary $\dot{J}^-(\mathcal{I}^+)$ of the causal past $J^-(\mathcal{I}^+)$ of future null infinity $\mathcal{I}^+$. The black hole region B is $B = M - J^-(\mathcal{I}^+)$ and the event horizon $H = \dot{J}^-(\mathcal{T}^+)$. This definition depends on the whole future behavior of the metric. There is a different sort of horizon, trapped horizon, which depends only on the properties of space-time on the surface $\Sigma(\tau)$ [2, 33].

We discussed the transition amplitude (propagator) between definite configurations of fields, $< h'', \phi'', \Sigma'' | h', \phi', \Sigma' >$. The transition amplitude from a state described by the wavefunction $\Psi^{in}[h', \phi']$ to a state $\Psi^{out}[h'', \phi'']$ reads

$$< \Psi^{out} | \Psi^{in} > =$$

$$\int \bar{\Psi}^{out}[h'', \phi''] < h'', \phi'', \Sigma'' | h', \phi', \Sigma' > \Psi^{in}[h', \phi'] \mathcal{D}h'\mathcal{D}\phi'\mathcal{D}h''\mathcal{D}\phi''.$$

Consider a family of asymptotically flat spacetimes. A wave function is a functional of the 3-geometry and the matter fields $\Psi[h_{ij}, \phi]$. It satisfies the Wheeler - De Witt equation

$$\mathcal{H}\Psi = 0$$

where $\mathcal{H}$ is the density of the Hamiltonian constraint.

Let ρ_Σ be the density operator of the Universe at the surface Σ. One defines the fine grained entropy

$$S^{\text{fine}}(\Sigma) = -\text{Tr}\, \rho_\Sigma \ln \rho_\Sigma$$

and correlation functions

$$f^{(s)}_{i_1 j_1 \ldots i_s j_s}(x_1, ..., y_s; \Sigma) = Tr[\rho_\Sigma \hat{g}_{i_1 j_1}(x_1)...\hat{\pi}_{i_s j_s}(y_s)]$$

where $\hat{h}_{ij}, \hat{\pi}_{ij}$ operators of metric and its canonically conjugate.

The correlation functions satisfy a system of equations in superspace. The equations are complicated. We can assume that there is a sort of unitary dynamics which preserves the fine grained entropy but there is no way to determine it. This is similar to the gas dynamics which was discussed in the previous section. But let us try to derive an analogue of the Boltzmann equation. We consider an approximation:

$$f^{(s)}_{i_1 j_1 \ldots i_s j_s}(x_1, \ldots, y_s; \Sigma) \to \prod_r f_{i_r j_r m_r n_r}(x_r, y_r; \Sigma), \quad \Sigma > \Sigma_0,$$

where

$$f_{i_r j_r m_r n_r}(x_r, y_r; \Sigma) = Tr[\rho_\Sigma \hat{g}_{i_r j_r}(x_r) \hat{\pi}_{m_r n_r}(y_r)]$$

This is similar to the Bogoliubov approximation (3). Quantum Boltzmann equation in quantum gravity will have the form

$$\frac{\delta f}{\delta \sigma} = J(f)$$

where $f = f_{i_r j_r m_r n_r}(x_r, y_r; \Sigma)$ and $\sigma = (x, y, g, \pi, \Sigma)$. The problem is to determine an explicit form of the functional $J(f)$.

The coarse grained entropy is

$$S^{\text{coarse}}(\Sigma) = - \int tr f(\cdot; \Sigma) \ln f(\cdot; \Sigma)$$

where an appropriate normalization for f is assumed.

Further, if we make also the semiclassical approximation, or assume the coherent pure states, then, in principle, we could get the classical Einstein equations for the metric $g_{\mu\nu}(x)$. However the form of classical equations depend on the chosen state.

For classical gravity the known laws of black hole thermodynamics are valid. In this case the entropy, which is proportional to the area of horizon, is a coarse grained entropy. This entropy increases during the classical evolution, so one gets a loss of the coarse grained information.

We speculate that the coarse grained entropy which is obtained in the Bogoliubov approximation increases in the classical regime. Note that the Hamiltonian in mini-superspace [28] is an N- particle hamiltonian but it has a form more complicated then the Hamiltonian for gases. One can try to study this problem for the Matrix model [34]. Note however that the irreversibility problem is a rather difficult problem even for such a simple and well studied system as quantum baker's map [35].

On later times the evaporation is the only relevant but slow process (quantum regime). Then the black hole looses matter and its coarse grained entropy decreases. This matter will go to the bath, so its coarse grained entropy is expected to increase such that the total coarse grained entropy also increases.

CONCLUSIONS

The information paradox of the black hole problem has bothered scientists since the discovery of the evaporation process. It has often not been realized, however, that "the" entropy of a system does not exist. The thermodynamic entropy of a system in contact with a bath at temperature T is defined in terms of added heat dQ as $dS = dQ/T$. The "classical intuition" of entropy arose when Bolztmann showed that this thermodynamic entropy agrees with a measure of disorder, namely the logarithm of the number of relevant states. For a closed system, this quantity cannot decrease. On the other hand, it is known in quantum mechanics that the von Neumann entropy *of a closed system* is a constant due to unitary motion. This is the quantum analog of the classical fine grained entropy, which is also conserved in time.

A second complicating factor it that the setup for black hole thermodynamics is one of systems far from equilibrium [19]. Thus the evaporation process is a problem of thermodynamics far from equilibrium, to which Gibbsian thermodynamics does not apply and for which, in general, few tools are available.

The problem of black hole evaporation is one of the field of *quantum thermodynamics*: the target system is small (the hole), but the bath is large (the rest of the Universe) and also the work source is large (here it would stand for the incoming matter, or work done externally on the hole), see e.g.[24]. In this field one can imagine a condensed matter analog where, starting from a ground state, work is from the outside put into a certain degree of freedom

(" growing of a toy black hole"), that is later taken out (its disappearance). In this process, not all work can be recovered due to Thomson's formulation of the second law: cyclic processes done on an equilibrium system (here: in its ground state) cannot yield work, and typically will cost work. This work will end up as phonons running away in the infinite condensed matter bath, in the very same way as matter and photons evaporated from the black hole will run in the otherwise empty, infinite Universe. We plan to discuss this setup in future, considering the separate entropies in detail [21]. The recent work on a realistic quantum measurement [36] shows that measurement problems are probably disconnected from black holes issues.

There is a little hope that quantum gravity or string theory could help to get an insight into the fundamental irreversibility problem until the further progress in the considerations of simple models will be achieved. However we should remind that Boltzmann has predicted the cosmological Big Bang just from the consideration of the irreversibility problem [13]. Not only the black hole problem but also the recent discovery of the cosmological acceleration and the mystery of dark energy indicates, it seems, to the necessity of the unified treatment of the basic problems in cosmology, quantum gravity/string theory, high energy physics, statistical physics and quantum information theory.

As to the black hole problem itself, we have outlined how to derive an equivalent of the Boltzmann entropy for gravitation. Also this subject deserves further attention. One outstanding question is to show that, taken together with the coarse grained bath entropy, it is non-decreasing.

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